Average-cost Pricing and Dynamic Selection Incentives in the Hospital Sector

Mathias Kifmann  Luigi Siciliani

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Abstract
This study investigates hospitals’ dynamic incentives to select patients when hospitals are remunerated according to a prospective payment system of the DRG type. Given that prices typically reflect past average costs, we use a discrete-time dynamic framework. Patients differ in severity within a DRG. Providers are to some extent altruistic. For low altruism, a downward spiral of prices is possible which induces hospitals to focus on low-severity cases. For high altruism, dynamic price adjustment depends on relation between patients’ severity and benefit. In a steady state, DRG prices are unlikely to give optimal incentives to treat patients.

Keywords: Hospitals; DRGs; selection; severity.
JEL Classification: I11, I18, L13, L44.

Mathias Kifmann
Department of Socioeconomics and Hamburg Center for Health Economics
Universität Hamburg
Esplanade 36
20354 Hamburg
Germany
mathias.kifmann@wiso.uni-hamburg.de

Luigi Siciliani
Department of Economics and Centre for Health Economics
University of York
Heslington
York YO10 5DD
UK
luigi.siciliani@york.ac.uk

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1 Introduction

Prospective payment systems based on Diagnosis Related Groups (DRG) are now in use worldwide to finance hospitals. They were first introduced in the US to replace cost-based reimbursement rules which were associated with high health care expenditure. In most cases, a fixed tariff is paid for each patient, creating strong incentives to contain costs.\(^1\) Cost efficiency, however, is only one aspect of the design of payment systems. Prospective payment systems also influence how and which patients are treated. One problem is that hospitals may save on the quantity and quality of services (Ellis and McGuire 1986, 1990), in particular if demand is not responsive to quality.\(^2\) This study focuses on the incentives for patient selection created by DRG-based payment. In particular, hospitals can have a financial incentive to ‘dump’ patients, ie to avoid patients whose care requires high spending (Dranove, 1987; Newhouse, 1983).\(^3\)

So far, the incentives to select patients have only been analysed in a static framework in which the DRG price is administratively set.\(^4\) This neglects that DRG prices are based on past average costs of all or a selection of hospitals where the time lag to measure costs is one or two years. This calculation principle underlies the calculation of DRGs weights and their conversion into monetary values in the US Medicare system and in most European countries (see Cots et al, 2011, for a survey).\(^5\) As we show in our paper, considering this pricing rule can have a profound effect on the incentives to select patients over time. In particular, it is not necessarily the case that high-severity patients are dumped. We show that prices can also evolve in such a way that overtreatment of both low- and high-severity patients can arise.

Our analysis is based on three key assumptions. Firstly, we assume that providers are, at least to some extent, altruistic.\(^6\) Secondly, we start from the premise that hospitals are able to select

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\(^1\)Hospitals receive additional payments only for ‘outliers’ for exceptionally high expenses.

\(^2\)If demand is responsive to quality, hospitals with poor quality will be penalised by lower revenues (Chalkley and Malcomson, 1998a; Ma, 1994). For other dimensions of quality, however, further measures may be necessary, in particular monitoring of standards. Furthermore, cost sharing can be useful in enhancing quality (Chalkley and Malcomson, 1998b).

\(^3\)Varying the intensity of service with the severity of a patient is a related strategy (Allen and Gertler, 1991; Ellis 1998). Hospital may try to overprovide services to low-severity patients (also known as ‘creaming’) and to underprovide services to high-severity patients (‘skimming’).


\(^5\)Shleifer (1985) argues that this average-cost rule is essential in creating ‘yardstick competition’ between hospitals. It gives each hospital the incentive to invest efficiently in cost reduction. Prices will then fall to the efficient level, mimicking a competitive market. In contrast to our analysis, however, Shleifer does not consider that hospitals can also influence their cost by selecting patients.

\(^6\)In the theoretical health economics literature, it has long been recognised that providers (doctors and nurses)
patients. For example, patients can be rejected by stating that a treatment has little chance of success or by pretending to have no more capacities.Providers can exploit the asymmetry of information which characterizes the doctor-patient relation (Arrow, 1963; McGuire, 2000).

Thirdly, we assume that DRGs do not perfectly capture the expected cost of each patient as perceived by providers. In particular, asymmetric information between payer and provider puts constraints on the ability to refine DRGs.

In our theoretical contribution, we model patients’ heterogeneity by assuming that they differ in severity within a DRG and that more severe patients are more costly. Providers balance the benefit from the DRG price and from the altruistic motive against the cost of treating a patient. Only patients will be treated for whom these benefits exceed cost. This defines a ‘marginal patient’ such that the benefit from the DRG price and from the altruistic motive is equal to the cost of treating this patient. Price dynamics arise if the average cost resulting from the providers’ treatment decisions differs from the DRG price. Then the price will be adjusted in the next period, giving new incentives for providers to treat patients.

Our key result is that selection incentives can be reinforced by these pricing dynamics. Consider first the case in which hospitals are not altruistic and treat patients with different severity. The marginal patient is characterized by cost equal to the DRG price. Then average cost must be lower and the DRG price must fall in the next period. This induces a ‘spiral to the bottom’ with price decreasing over time. Altruistic hospitals, by contrast, are willing to treat patients which cause losses. Then average cost can be above the DRG price. In this case altruism has as a double effect on patients’ selection. Not only does higher altruism lead to more patients being treated but with a delay also increases the tariff which further increases the number of patients treated. In equilibrium, it is possible that all patients are treated. In
addition to altruism, the relation between patients’ severity and benefit influences the price dynamics. We also show that prices always converge to an equilibrium DRG price.

After investigating hospital incentives under current payment systems, we conduct a welfare analysis. We compare the patients selected under DRG pricing and corresponding severity with the first-best allocation. Here it is crucial whether patients’ benefit decreases or increases with severity. For instance, if a treatment requires a good physical constitution to recover, patients’ benefit is likely to decrease with severity. This suggests that patients below a critical severity threshold should receive treatment. If altruism is sufficiently low, DRG pricing may lead to too few patients being treated. But already for moderate levels of altruism, too many patients can be treated. Higher severity can also be associated with higher patients’ benefit, for example, when higher severity causes higher pain without treatment. Then in the presence of low altruism, DRG pricing may give completely the wrong incentives. The provider has incentives to treat low-severity patients while it would be optimal from a first-best perspective to treat those with high severity. With higher levels of altruism, high-severity patients will be treated. However, it is still the case that low-severity patients receive the treatment when they should not.

We proceed as follows. In Section 2, we present the model and derive the dynamics of DRG pricing and DRG equilibrium prices. Section 3 characterises the first-best allocation and evaluates the outcome of the DRG system. Section 4 concludes.

2 The Model

We focus on one DRG. There are \( N \) identical hospitals that provide treatment to patients. In each hospital, patients with a given diagnosis differ in severity of illness \( s \) which is distributed over the support \([s, \bar{s}]\) with density function \( f(s) \) and cumulative distribution function \( F(s) \). We assume that \( f(s) \) does not vary with time \( t \). The expected cost of treating a patient with severity \( s \) is \( c(s) \), and is increasing in severity, \( c'(s) > 0 \).\(^{10}\) Patient’s benefit is given by \( b(s) \). We assume that benefit is positive and can increase or decrease with severity, \( b'(s) \geq 0 \). For

\(^{10}\)We do not model explicitly incentives to contain costs, e.g., by introducing a cost-containment effort variable as in some of the studies cited above. This is because we focus on a purely prospective DRG system. The provider is residual claimant and will choose optimal cost-containment effort. Therefore, adding such a variable would make the presentation more complex without adding significant additional insights.
example, $b'(s) > 0$ captures the case when a higher severity causes higher pain without treatment. $b'(s) < 0$, by contrast, can describe a treatment which requires a good physical constitution to recover. Patients’ severity is observed by providers but not by the payer.

Patients have a passive role and accept to undertake the treatment if recommended by the provider. We adopt a model in discrete time. Hospitals receive a DRG price in each period $t$ which is denoted by $p_t$ for patients with the same diagnosis. The DRG price is set according to the average cost of all patients who are treated with the same diagnosis in the previous period.

Providers are partially altruistic. Altruism (or motivation) is captured by the parameter $\alpha$ with $0 \leq \alpha \leq 1$. The provider’s altruistic gain from each patient treatment is $\alpha b(s)$. A selfish provider ($\alpha = 0$) cares only about the difference between revenues and costs. In each period the provider treats the patient if $p_t \geq c(s) - \alpha b(s)$, ie if the price at least covers the ‘net cost’, defined as the expected cost minus the altruistic gain. We assume that the net cost is either increasing or decreasing over the severity support $[s, \bar{s}]$. In an interior solution, the marginal treated patient has severity $s_t$ which is implicitly defined by

$$p_t = c(s_t) - \alpha b(s_t), \quad (1)$$

ie the DRG tariff is equal to the marginal net cost. Whether the provider treats low-severity or high-severity patients under this interior solution depends on specific assumptions about altruism, the benefit and the cost function.

In the following we distinguish two different scenarios, increasing and decreasing net cost with severity:

(I) Net cost increasing with severity: $c'(s) - \alpha b'(s) > 0$ in $[s, \bar{s}]$. In such case, low-severity patients are treated in an interior solution. This scenario always arises if patient’s benefit decreases with severity, $b'(s) < 0$: low-severity patients have lower costs and higher benefits, making these patients attractive for providers both for profit and altruistic reasons.

Hence, providers treat patients with severity below or equal to $s_t$ in period $t$ (defined in

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11 We interpret the parameter $\alpha$ as altruism. More generally, this parameter could encompass other sources of motivation, including reputational concerns, peer pressure, and fear of malpractice suits.

12 These scenarios are not exhaustive as a non-monotonic relation between net cost and severity is also possible. To keep the exposition focused, we concentrate on the two clear-cut cases.
eq. (1)). This scenario can still arise if patient’s benefit does not vary with severity, or if it increases with severity and either altruism or the marginal benefit from higher severity is sufficiently small.

(D) Net cost decreasing with severity: $c'(s) - \alpha b'(s) < 0$ in $[s, \pi]$. In such case, high-severity patients are treated in an interior solution. This scenario obtains when patients’ benefit increases with severity and $\alpha b'(s)$ is sufficiently high: although high-severity patients are more costly, their benefit from treatment (weighted by the degree of altruism) is sufficiently high that providers have an incentive to treat all patients with severity above $s_t$ in period $t$ despite the fact that more severity implies higher treatment costs. This scenario requires positive altruism.

In the following we discuss the two different scenarios separately, first starting by the case where providers have an incentive to focus on low-severity patients.

2.1 Scenario I: net cost increasing with severity

In each period $t$ the DRG price is set according to the average cost of the patient treated in the previous period. In Scenario I the net costs is increasing with severity (with $c'(s) - \alpha b'(s) > 0$ in $[s, \pi]$), and therefore providers select low-severity patients if they do not treat all patients. The DRG price in period $t$ is equal to

$$p_t = \frac{\int_{s_{t-1}}^{s_{t-1}(p_{t-1})} c(s)f(s)ds}{F(s_{t-1}(p_{t-1}))} \equiv g(p_{t-1}), \quad (2)$$

where $F(s_{t-1}(p_{t-1})) = \int_{s_{t-1}(p_{t-1})}^{s_{t-1}(p_{t-1})} f(s)ds$ is the number of patients treated in the previous period and $s_{t-1}(p_{t-1})$ is the marginal severity in the previous period $t - 1$. It is defined by $p_{t-1} = c(s_{t-1}) - \alpha b(s_{t-1})$ and determined by the price in the previous period $p_{t-1}$. The pricing rule is therefore recursive since the price in period $t$ depends on the price in period $t - 1$, which is captured more succinctly by the function $p_t = g(p_{t-1})$.

To examine the price dynamics, we define

$$p_{min}^f \equiv c(\underline{s}) - \alpha b(\underline{s}) \quad (3)$$


as the minimum price under scenario I which induces providers to treat patients, which is equal
to the net cost of the patient with lowest severity. If the price is below this level, ie \( p_{t-1} < p_{I_{\text{min}}} \),
then nobody will be treated. In the following, we assume an initial price above the net cost
of patients with lowest severity, ie \( p_0 \geq p_{I_{\text{min}}} \) in period 0, so that some patients are treated in
the initial period. The assumption that the initial price is above such level is intuitive and not
restrictive. It implies that the initial price is above a level, which is below the minimum cost.\(^{13}\)

Similarly, we define
\[
p_{I_{\text{all}}} \equiv c(\overline{s}) - \alpha b(\overline{s}) \tag{4}
\]
as the price under scenario I above which providers treat all patients. Such price is equal to the
net cost of the patient with highest severity. If the price is above such level, ie \( p_{t-1} \geq p_{I_{\text{all}}} \), then
equation (2) implies that the price in the following period is equal to the average cost when all
patients are treated \( c' \equiv \int_{s_0}^{s_{t-1}} c(s) f(s) \text{d}s \). Increasing net costs, \( c'(s) - \alpha b'(s) > 0 \), imply that the
net cost of the patient with highest severity is higher than the net cost of the patient with lowest
severity so that \( p_{I_{\text{min}}} < p_{I_{\text{all}}} \).

For prices \( p_{t-1} \) in the intermediate range \([p_{I_{\text{min}}}, p_{I_{\text{all}}})\), the price dynamics follows equation (2)
which is a non-linear first-order difference equation. Price dynamics can therefore be summed
up in
\[
p_t(p_{t-1}) = \begin{cases} 
  c' & \text{if } p_{t-1} \geq p_{I_{\text{all}}} \\
  g(p_{t-1}) & \text{if } p_{I_{\text{min}}} \leq p_{t-1} < p_{I_{\text{all}}}. 
\end{cases} \tag{5}
\]

For intermediate prices \( p_{I_{\text{min}}} \leq p_{t-1} < p_{I_{\text{all}}} \), we can implicitly differentiate the function
\( g(p_{t-1}) \) defined by (2) and obtain
\[
\frac{dp_t}{dp_{t-1}} = f(s_{t-1}(p_{t-1})) \left[ c(s_{t-1}(p_{t-1})) - \frac{\int_{s_0}^{s_{t-1}(p_{t-1})} c(s) f(s) \text{d}s}{F(s_{t-1}(p_{t-1}))} \right] \frac{\partial s_{t-1}}{\partial p_{t-1}} > 0. \tag{6}
\]

A higher price in a period increases the price in the next period. This is due to \( \partial s_{t-1}/\partial p_{t-1} = 1/[c'(s) - \alpha b'(s)] > 0 \): a higher price implies a higher marginal severity. Providers are therefore
willing to treat more costly patients. This increases average cost and therefore the price in the

\(^{13}\)An alternative stronger but still plausible assumption is that the price is between the lowest cost and the
highest cost, which seems consistent with average-cost pricing rules, ie they are bounded to be between the
minimum and the maximum cost.
Price dynamics can be examined in a phase diagram in which the variables $p_t$ and $p_{t-1}$ are plotted against each other. If $p_t(p_{t-1})$ is above the $45^\circ$ line, the price in $t-1$ is below average cost and the price increases. Conversely, the price in $t-1$ is above average cost where $p_t(p_{t-1})$ is below the $45^\circ$ line and the prices falls. Equilibria are given by the intersections of $p_t(p_{t-1})$ with the $45^\circ$ line.

Suppose that the initial price is set at the minimum, $p_{t_{\text{min}}} = c(s) - \alpha b(s)$, ie the net cost of the patient with the lowest severity. Then providers only treat patients with the lowest severity and in the following period the price will be equal to $p_t(p_{t_{\text{min}}}) = c(s)$. Therefore, $p_t(p_{t_{\text{min}}})$ lies above the $45^\circ$ line for any strictly positive level of altruism.$^{15}$ In the absence of altruism, we have $p_t(p_{t_{\text{min}}}) = c(s) = p_{t_{\text{min}}}$ which is on the $45^\circ$ line. Furthermore, note that prices are bounded from above by $c^\mu$. Thus, for high prices $p_{t-1}$, the function $p_t(p_{t-1})$ is below the $45^\circ$ line and prices must be falling.

Let us first consider in Figure 1.a the special case where altruism is equal to zero and providers only maximise profits. In this case, the price function evaluated at the lowest price $p_t(p_{t_{\text{min}}})$ is on the $45^\circ$ line. For higher prices, $p_t(p_{t-1})$ must be below the $45^\circ$ line. This is because profit-maximising providers treat patients only up to the point where price is weakly above the cost, $p_{t-1} = c(s_{t-1})$. Average cost of these patients must be below $c(s_{t-1})$ because cost increases with severity, leading to a lower price in the next period. Thus, there is a unique equilibrium $p^* = p_{t_{\text{min}}} = c(s)$ in which only the lowest severity patients are treated. Starting from a price above the equilibrium leads to a downward spiral. This is shown by the arrows in Figure 1.a.

The DRG system induces dumping which in the following periods reduces the average cost of treatment and therefore the DRG price, leading to increased incentives for dumping. This spiral comes to a halt at the equilibrium price, implying an almost complete unraveling of the market. This is a rather negative result. It implies that DRG pricing has the potential to generate a dynamics towards the ‘bottom’. Given an initial (high) price, providers have an incentive to select patients, and treat only those with low severity. This in turn implies that the following

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$^{14}$The term in the square brackets in equation (6) is positive since the cost of the marginal patient is always higher than the average cost (recall that net cost is increasing in severity).

$^{15}$If $p_{t_{\text{min}}}$ could theoretically be negative. However, we must also have $p_t(0) > 0$ in this case because for a price of zero, patients are (weakly) treated, leading to positive costs and therefore to a positive price in the next period.
Figure 1: Altruism and Price Dynamics
period the price will be lower, which will induce even further selection and reduce marginal severity, and so on.

Next, we turn to altruistic providers. We first examine cases in which the equilibrium is unique (Figures 1.b and 1.c) before we turn to the possibility of multiple equilibria (Figure 2). Figure 1.b shows a setting in which the location of the price function $p_t(p_{alt})$ lies below the 45° line. There is a unique equilibrium with price $p^*$ between $p_{min}^I$ and $p_{all}^I$. Again, a downward spiral arises for an initial price $p_0 > p^*$. For an initial price below $p^*$, however, prices increase over time until they come to a halt at the equilibrium price. The outcome is therefore not as extreme as in Figure 1.a. However, some selection takes place.

The downward spiral in Figure 1.b could reflect the price and selection dynamics when cost reimbursement rules are replaced with a DRG system. Under the old regime, hospitals have strong incentives to admit patients since expensive patients were simply reimbursed. Furthermore, incentives for cost efficiency are low. This implies high average cost and therefore a high initial DRG price. Thus, falling prices and costs cannot only be explained by higher cost-containment incentives of the DRG system but also by selection incentives that get reinforced over time.

Figure 1.b shows that with altruism, however, an upward price spiral is also possible. For an initial price $p_0 < p^*$, prices increase over time until they reach the equilibrium price. To understand these dynamics, consider the initial price $p_0 = p_{min}^I < c(s)$. Altruistic providers are willing to treat the lowest-severity patient even though the price is below cost which corresponds to average cost in this case. In the next period, the price must increase. Thus, altruism has as a double effect on patients’ selection. Not only does it lead to more patients being treated but with a delay also increases the tariff which further increases the number of patients treated.

In Figure 1.c, altruism is high. Providers are willing to treat all patients for a low price $p_{alt}^I$ (see equation (4)). We have $p_t(p_{alt}^I) = c^u > p_{alt}^I$ and $p_t(p_{alt}^I)$ is therefore above the 45° line. In this case, only increasing prices are possible leading to the equilibrium price $p^* = c^u$. Starting from a lower price leads to an upward spiral. Since providers’ altruism is high, they are willing to treat patients whose costs are larger than the DRG price. This in turn implies a higher tariff in the next period which further encourages providers to treat high-cost patients.
The degree of altruism is therefore critical in characterising whether prices are decreasing or increasing over time. As can be seen from the figures, it is crucial whether the kink in the function $p_t(p_{t-1})$ is above or below the $45^\circ$ line. It is below if $p_t(p_{\text{all}}^I) = c^\mu < p_{\text{all}}^I$, i.e. when the average cost of treating all patients is lower than the net cost of treating the patient with highest severity. We therefore find that decreasing prices are possible (for a sufficiently high initial price) if and only if

$$p_t(p_{\text{all}}^I) = c^\mu < p_{\text{all}}^I = c(\bar{s}) - \alpha b(\bar{s}) \iff \alpha < \hat{\alpha} \equiv \frac{c(\bar{s}) - c^\mu}{b(\bar{s})} > 0,$$

where $\hat{\alpha}$ is critical level of altruism below which not all patients are treated in equilibrium and prices can be decreasing if the initial price is sufficiently high. For altruism above $\hat{\alpha}$, prices can only increase until they reach the highest possible level, i.e. the average cost of all patients. In Figure 1.a and 1.b, altruism is below $\hat{\alpha}$. Figure 1.c shows the case of high altruism $\alpha \geq \hat{\alpha}$. The unique stable equilibrium is given by $p^* = c^\mu$ and everybody is treated. This suggests the intuitive result that higher altruism induces more treatment which we examine in more detail below.
So far, we have only considered cases with an unique equilibrium. For positive altruism, also multiple equilibria are possible. Figure 2 shows an example. The function $p_t(p_{t-1})$ intersects the 45° line three times, leading to two stable (A and C) and one instable equilibrium (B). More than three equilibria are also possible, with the total number being odd. Since $p_t(p_{t-1})$ starts above the 45° line and is flat for $p_{t-1} \geq p_{all}^I$, there is always one more stable equilibrium than unstable equilibrium. With multiple equilibria, both decreasing and increasing prices are possible depending on the initial price.\footnote{Note that the slope of $p_t(p_{t-1})$ depends on $\frac{\int_{s_t-1}^{s_{t-1}} c(s)f(s)ds}{F(s_{t-1}^I(p_{t-1}))}$. Therefore, the slope may be low when the mass of patients for a given severity is low.}

To summarise

**Proposition 1** If the net cost is increasing in severity, there is always at least one stable equilibrium. Altruistic providers treat patients up to a severity level $s^*$ in $[s, \bar{s}]$ in this equilibrium. For low altruism, only a share of potential patients will be treated. Depending on the initial price, decreasing or increasing prices are possible. Non-altruistic providers only treat patients with the lowest severity $s$. If altruism is sufficiently high, then treating all patients is a stable equilibrium. If this is the unique equilibrium, prices can only be increasing.

Interior equilibria are described by the following two equations for equilibrium price and severity, $p^*$ and $s^*$:

\begin{align*}
    p^* &= \frac{\int_{s}^{s^*(p^*)} c(s)f(s)ds}{F(s^*(p^*))}, \\
    p^* &= c(s^*) - \alpha b(s^*). \\
\end{align*}

Equation (8) is simply the DRG pricing rule demanding that price equals average cost. Equation (9) characterises the equilibrium selection decision by providers. It states that in an interior equilibrium, price must be equal to the net cost of the marginal patient. Using the right-hand sides, we can further characterize the optimal marginal severity $s^*$ as

\[ \alpha b(s^*) = c(s^*) - \frac{\int_{s}^{s^*} c(s)f(s)ds}{F(s^*)}. \]

In equilibrium, the marginal severity below which patients are treated is defined such that the
benefit from treatment (weighted by altruism) is equal to difference between the marginal cost and the average cost. Since the cost is increasing in severity, the marginal cost is always above the average cost (and the price) and the right-hand side of equation (10) is positive.

If providers are profit maximisers, then condition (10) is satisfied only if patients with lowest severity are treated so that marginal and average cost coincide. Any small positive level of altruism implies that patients above minimum severity are treated so that the cost of treating the marginal patient is higher than the average cost. Therefore, a corner solution (with $s^* = s$) cannot arise for positive altruism.

Condition (10) emphasises the critical role played by altruism. Differentiating (10) with respect to $\alpha$, we obtain that higher altruism leads to a higher marginal severity starting from a stable interior equilibrium and to a higher price: $\partial s^*/\partial \alpha > 0$ and $\partial p^*/\partial \alpha > 0$ (Proof in the Appendix). Higher altruism implies that providers are willing to treat more severe patients at the margin. This in turn implies higher cost which translates into a higher price.

**Proposition 2** In a stable interior equilibrium, the marginal benefit from treatment weighted by altruism equates the difference between the marginal and average cost of treatment. The price and the marginal equilibrium severity, below which patients are treated, increase with altruism.

The result that price increases with altruism is in sharp contrast with what obtained under optimal pricing rules where the regulator can design the optimal price as a function of the parameters at stake, including altruism (e.g., Ellis and McGuire, 1986). Under an optimal pricing rule, higher altruism generally implies a lower price. Intuitively, higher altruism implies that the provider needs less to be incentivised through prices. When prices are second best and determined by an average-cost rule, then higher altruism implies a higher price. Higher altruism implies a higher willingness to treat patients which ultimately is reflected into a higher average cost and therefore a higher price.

**An example**

Assume that benefits and costs are linear in severity: $b(s) = b_0 + b_1 s$, $c(s) = c_0 + c_1 s$ with $c_0 > 0$, $c_1 > 0$, $b_0 > 0$, and that patients’ severity is distributed over the support $[s, \bar{s}]$ with the uniform density function with a mass of one, $f(s) = 1$, (implying $\bar{s} - s = 1$). Under scenario I we have
\(c'(s) - ab'(s) > 0\) and therefore \(c_1 - ab_1 > 0\). In the Appendix, we show that the function \(g(p_t)\) is linear: \(g(p_t) = \frac{A}{2(c_1 - ab_1)} + \frac{c_1}{2(c_1 - ab_1)}p_{t-1}\) with \(A \equiv sc_1(c_1 - ab_1) + c_0c_1 + \alpha (c_1b_0 - 2b_1c_0)\). This implies that there is always a unique equilibrium for \(p_0 \geq p_{min}\). We find:

(a) For \(\alpha = 0\), we have \(p^* = c(s), s^* = \bar{s}\).

(b) For \(0 < \alpha < \hat{\alpha} = c_1/[2(b_0 + b_1\bar{s})]\), we have an interior equilibrium with \(p^* = \frac{A}{c_1 - 2ab_1}, s^* = \frac{sc_1 + 2ab_0}{c_1 - 2ab_1}\).

(c) For \(\alpha \geq \hat{\alpha}\), we have \(p^* = c^\mu, s^* = 0\).

The equilibria correspond to Figures 1.a to 1.c where to plot the figures we have used the following parameter values: \(b_1 = 0, b_0 = c_1 = 1, c_0 = 0.5, \bar{s} = 0\) and \(\alpha \in \{0, 0.25, 0.75\}\).

### 2.2 Scenario D: net cost decreasing with severity

We now turn to scenario D where the net cost of treating a patient decreases with severity: \(c'(s) - ab'(s) < 0\). For a given price, assuming they do not treat all patients, providers will treat high-severity patients. This case can arise only if patient’s benefit increases with severity \((b'(s) > 0)\) and altruism is sufficiently high.

Since now patients in the interval \([s_{t-1}, \bar{s}]\) are treated, the DRG price in period \(t\) is given by

\[
p_t = \frac{\int_{s_{t-1}(p_{t-1})}^{\bar{s}} c(s)f(s)\,ds}{1 - F(s_{t-1}(p_{t-1}))} \equiv h(p_{t-1}), \quad (11)
\]

where \(1 - F(s_{t-1}(p_{t-1})) = \int_{s_{t-1}(p_{t-1})}^{\bar{s}} f(s)\,ds\) is the number of patients treated and \(s_{t-1}(p_{t-1})\) is the marginal severity in the previous period \(t - 1\), defined by \(ab(s_{t-1}) + p_{t-1} = c(s_{t-1})\).

As above, we define the minimum price which induces providers to treat some patients. Because in this case providers have a preference for high-severity patients, the price in scenario D is given by

\[
p_{Dmin}^p \equiv c(\bar{s}) - ab(\bar{s}), \quad (12)
\]

which is the net cost of treating the patient with highest severity. Again, we assume that the initial price is above this level, \(p_0 \geq p_{min}^D\).
Furthermore, we define
\[ p^D_{\text{all}} \equiv c(s) - \alpha b(s) \] (13)
as the price above which providers treat all patients, which is equal to the net cost of treating the patient with lowest severity. The assumption \( c'(s) - \alpha b'(s) < 0 \) implies that \( p^D_{\text{min}} < p^D_{\text{all}} \). If \( p_{t-1} \geq p^D_{\text{all}} \), the price in the following period is equal to the average cost when all patients are treated, \( c^\mu = \int_s^s c(s) f(s) ds \).

For intermediate prices \( p_{t-1} \) in \([p^D_{\text{min}}, p^D_{\text{all}}]\), the price dynamics follow the non-linear first-order difference equation (11) which implicitly defines the function \( p_t = h(p_{t-1}) \). Price dynamics can therefore be summed up in

\[
p_t(p_{t-1}) = \begin{cases} 
    c^\mu & \text{if } p_{t-1} \geq p^D_{\text{all}} \\
    h(p_{t-1}) & \text{if } p^D_{\text{min}} \leq p_{t-1} < p^D_{\text{all}}.
\end{cases}
\] (14)

For intermediate prices \( p^D_{\text{min}} \leq p_{t-1} < p^D_{\text{all}} \), we can implicitly differentiate (11) and obtain

\[
\frac{dp_t}{dp_{t-1}} = -\frac{f(s_{t-1}(p_{t-1}))}{1 - F(s_{t-1}(p_{t-1}))} \left[ c(s_{t-1}(p_{t-1})) - \frac{\int_{s_{t-1}(p_{t-1})}^s c(s) f(s) ds}{1 - F(s_{t-1})} \right] \frac{\partial s_{t-1}}{\partial p_{t-1}} < 0. \] (15)

A higher price in a period decreases the price in the next period. This result is in stark contrast to the result obtained in the previous section (see equation (6)) and is due to \( \frac{\partial s_{t-1}}{\partial p_{t-1}} = 1/[c'(s) - \alpha b'(s)] < 0 \): a higher price now reduces the marginal severity. Providers are therefore willing to treat further patients who have higher net cost, but lower cost. This decreases the average cost and therefore the price in the next period.\(^{17}\)

In the phase diagram in Figure 3, the function \( p_t(p_{t-1}) \) is therefore falling in the range \([p^D_{\text{min}}, p^D_{\text{all}}]\) and then flat. Furthermore, \( p_t(p^D_{\text{all}}) = c^\mu \) is above the 45° line since \( p^D_{\text{all}} = c(s) - \alpha b(s) < c(s) < c^\mu \). The unique equilibrium is given by a price equal to the average cost, \( p^* = c^\mu \).

Dynamic price adjustments lead to a quick convergence to the equilibrium. Consider first initial prices \( p^D_{\text{min}} \leq p_0 < p^D_{\text{all}} \). In the next period the price rises above \( c^\mu \) because providers only treat high-severity patients. This price is above \( p^D_{\text{all}} = c(s) - \alpha b(s) \) which implies that all

\(^{17}\)The term in the square brackets in equation (15) is negative since the cost of the marginal patient is now lower than the average cost.
patients are treated in period 1. In period 2, the price is thus \( p_2 = c^\mu \) and the equilibrium is reached. Price convergence is even faster for an initial price above \( p^D_{\text{all}} \). Every patient is therefore treated in period 0. Thus, the price in the following period will be equal to the average cost \( p_t(p^D_{\text{all}}) = c^\mu \). The price will remain at this level and the equilibrium is reached already after one period.

**Proposition 3** If net cost is decreasing with severity, all patients are treated in the unique and stable equilibrium, and price is equal to the average cost of all patients. Depending on the initial price, the equilibrium price is reached in one period, or the price first rises and then falls to the equilibrium price in the second period.

Assume as in section 2.1 that benefit and cost is linear in severity and the density function is uniform. Under these assumptions scenario D arises if \( ab_1 > c_1 \) or, equivalently, \( \alpha > \tilde{\alpha} \equiv c_1/b_1 \).

In the Appendix, we show that the function \( h(p_{t-1}) \) is linear and given by \( h(p_{t-1}) = c_0 + c_1(s - \alpha b_0) \).

In equilibrium, we have a corner solution with \( s^* = \bar{s} \) and \( p^* = c_0 + c_1(\bar{s} + \bar{s})/2 \). To draw Figure 3, we have used the following parameter values: \( b_0 = b_1 = 2 \), \( c_1 = 1 \), \( c_0 = 1.5 \), \( \bar{s} = 0 \) and \( \alpha = 0.6 \).
2.3 Price dynamics and altruism

Our analysis has shown that price dynamics depend crucially on altruism. Low altruism makes a downward spiral likely, in particular if the system starts with a high initial price. With high altruism, the picture is more complex. An upward price spiral is now possible in Scenario I. In Scenario D, which requires both high altruism and patient benefit increasing in severity, the price adjustment can be non-monotonous. Prices can fall after an initial price rise. In contrast to the case of low altruism, this price fall is accompanied by an increase in volume as providers extend treatment to low-severity patients.

3 A Normative Analysis

From a policy perspective, the crucial question is whether price adjustments over time give providers the incentive to treat the “right” patients. This requires a normative analysis. In the following, we start from the premise that patients should be treated when benefits are (weakly) above costs: \( b(s) \geq c(s) \) for \( s \) over the support \([s_\text{L}, s_\text{H}]\).\(^{18}\) Again, we distinguish between two cases.

(L) **Low-severity patient should be treated in an interior solution.** Suppose that patient benefit is either decreasing with severity or increasing with severity but less steeply than cost: \( c'(s) - b'(s) > 0 \). Assuming an interior solution, the marginal patient is such that \( b(s^f) = c(s^f) \) and patients with severity \( s \) below \( s^f \) should receive treatment and those with severity above \( s^f \) should not.

(H) **High-severity patient should be treated in an interior solution.** Suppose that patients benefit increases with severity more quickly than costs: \( c'(s) - b'(s) < 0 \). Then the marginal patient is still characterised by \( b(s^f) = c(s^f) \) but patients with severity above \( s^f \) should now be treated. If benefit is higher than cost for all patients over the support \([s_\text{L}, s_\text{H}]\), then it is optimal to treat all patients.

\(^{18}\)We therefore do not consider provider benefit \( \alpha b(s) \) for the optimal treatment decision. In this, we follow Chalkley and Malcomson (1998b) who oppose a double counting of treatment benefits in social welfare calculations.
Scenarios L and H relate in the following way to scenarios I and D in the last section:

- Scenario L with \( c'(s) - b'(s) > 0 \) implies that provider’s scenario I with \( c'(s) - \alpha b'(s) > 0 \) holds for any \( \alpha \in [0,1] \). Therefore, under scenario L we can ignore provider’s scenario D because the latter never arises (since \( c'(s) - \alpha b'(s) < 0 \) never holds).

- Scenario H with \( c'(s) - b'(s) < 0 \) arises only if \( b'(s) \) is positive and sufficiently steep. Here, on the provider’s side we need to distinguish between scenario I and D. Provider’s scenario I arises when low altruism implies \( c'(s) - \alpha b'(s) > 0 \) and scenario D when high altruism causes \( c'(s) - \alpha b'(s) < 0 \).

3.1 Scenario L: \( c'(s) - b'(s) > 0 \). It is optimal to treat low-severity patients

We start by considering scenario L where it is optimal to treat low severity patients. If \( c(\overline{s}) - b(\overline{s}) > 0 \) and \( c(\underline{s}) - b(\underline{s}) < 0 \), an interior solution arises and it is optimal to treat patients with severity \( s \in [\underline{s}, s_f] \). Recall that if an interior solution arises (for \( \alpha < \hat{\alpha} \)) in provider’s scenario I, then the optimal provider’s equilibrium severity (see equation (10)) is characterised by

\[
\alpha b(s^*) + \frac{\int_{\underline{s}}^{s^*} c(s) f(s) ds}{F(s^*)} = c(s^*). \tag{10}
\]

When altruism is zero, only patients with lowest severity are treated and \( s^* = \underline{s} < s_f \). Equilibrium severity \( s^* \) increases with altruism, but as long as \( s^* < s_f \) too few patients are treated. It would be optimal from a welfare perspective to treat patients with severities in \((s^*; s_f]\) as well (see Figure 4, case (a)). At the opposite side of the spectrum, if altruism is high, too many patients are treated (see Figure 4, case (b)). Comparing the first-best severity characterized by \( b(s_f) = c(s_f) \) with condition (10), this must be the case for \( \alpha = 1 \). There are some patients with high severity that should not be treated but instead are.

The level of altruism \( \hat{\alpha} \) where \( s^* \) corresponds to \( s_f \) can be determined by using \( b(s_f) = c(s_f) \) and condition (10). We obtain

\[
\hat{\alpha} = 1 - \frac{\int_{\underline{s}}^{s_f} c(s) f(s) ds}{F(s_f)} \frac{1}{c(s_f)} < 1.
\]
The threshold is equal to one (the highest level of altruism) minus the ratio of the average and marginal cost of treatment when evaluated at the first-best level. The ratio between the average and marginal cost is always below one since the marginal cost is increasing and low-severity patients are treated. Undertreatment arises for $\alpha < \tilde{\alpha}$, overtreatment for $\alpha > \tilde{\alpha}$. Only by chance will providers implement the first-best solution.

To summarise:

**Proposition 4** Suppose that in the first-best solution it is optimal to treat low-severity patients $(c'(s) - b'(s) > 0, c(\overline{s}) - b(\overline{s}) > 0$ and $c(\underline{s}) - b(\underline{s}) < 0)$. Then,

- too few patient are treated for sufficiently low altruism $(\alpha < \tilde{\alpha} = 1 - \frac{AC(\underline{s}^{f})}{c(\underline{s}^{f})} < 1)$;
- too many patients are treated for sufficiently high altruism $(\tilde{\alpha} < \alpha < 1)$.
Again assume that benefit and cost is linear in severity, the density function is uniform and that the provider has an interior solution. Scenario L arises if $c_1 > b_1$ and $b_0 > c_0$, so that

\[ s^L = \frac{b_0 - c_0}{c_1 - b_1}, \quad s^* = \frac{sc_1 + 2\alpha b_0}{c_1 - 2\alpha b_1} \quad \text{and} \quad \tilde{\alpha} = \frac{c_1(b_0 - c_0 + s(b_0 - c_1))}{2(c_1 b_0 - b_1 c_0)}. \]

The solution is plotted in Figure 4 under the following parameter values: $b_1 = 0$, $b_0 = c_1 = 1$, $c_0 = 0.5$, $\alpha = 0$ which implies $s^L = 0.5$ and $s^* = 2\alpha$. In Figure 4, case (a) relies on $\alpha = 0.2$, in case (b) $\alpha = 0.4$.

Our results also extend to corner solutions. First, consider that it is optimal to treat all patients. A sufficient assumption is $c(s) - b(s) < 0$. Then undertreatment will arise if altruism is below $\tilde{\alpha}$. It is also possible that a DRG is active even though nobody should be treated. This case applies if $c(s) - b(s) > 0$. Then individuals will nevertheless be treated if the initial price is sufficiently high, ie above $p_{min}^L = c(s) - \alpha b(s)$.

### 3.2 Scenario H: $c'(s) - b'(s) < 0$. It is optimal to treat high-severity patients

In this case it is optimal in the first-best solution to treat patients with high severity, ie patients with severity above $s^L$. If $c(s) - b(s) > 0$ and $c(\bar{s}) - b(\bar{s}) < 0$, an interior solution arises and it is optimal to treat patients with severity $s \in [s^L, \bar{s}]$.

Recall that this case arises only if $b'(s)$ is positive and sufficiently steep. Here, on the provider’s side we need to distinguish between scenario I and D. Provider’s scenario I arises when $c'(s) - \alpha b'(s) > 0$ and scenario D when $c'(s) - \alpha b'(s) < 0$. For $\alpha = 1$ we have that $c'(s) - \alpha b'(s) = c'(s) - b'(s) < 0$, and for $\alpha = 0$ we have that $c'(s) - \alpha b'(s) = c'(s) > 0$. Since $c'(s) - \alpha b'(s)$ is decreasing in $\alpha$ (under scenario H $b'(s)$ has to be positive), there is a critical level of altruism $\tilde{\alpha} \equiv c'(s)/b'(s)$ such that for $\alpha < \tilde{\alpha}$ scenario I arises and above which scenario D arises.

For scenario I, we also need to consider whether altruism is below or above the threshold $\hat{\alpha}$ which causes the provider to treat all patients (see equation (7)). Then, for $\alpha < \min\{\tilde{\alpha}; \hat{\alpha}\}$, scenario I arises and only low severity-patients are treated while it is optimal to treat high-severity patients. This is a worrying case since the DRG system gives the wrong incentives. In Figure 4 case (c), this result is particularly marked as the incentives are completely wrong. In case (d), for higher altruism, at least some patients are treated who should be treated.
For $\alpha > \min\{\overrightarrow{\alpha}; \hat{\alpha}\}$ all patients are treated and the DRG system induces overtreatment as in Figure 4 case (e). The intuition is that altruism is so strong that all are treated ($\alpha > \hat{\alpha}$ in Scenario I) or that DRG pricing based on the highest-cost types makes it attractive to treat low-cost types as well ($\alpha > \overrightarrow{\alpha}$ and thus Scenario D).

**Proposition 5** Suppose that in the first-best solution it is optimal to treat high-severity patients ($b'(s) > 0; c'(s) - b'(s) < 0$ and $c(s) - b(s) > 0$). Then,

- for sufficiently low altruism ($\alpha < \min\{\overrightarrow{\alpha}; \hat{\alpha}\}$) there are some patients with low severity that are treated although benefits are below costs;

- too many patients are treated for sufficiently high altruism ($\alpha > \min\{\overrightarrow{\alpha}; \hat{\alpha}\}$).

Returning to our example from section 2.1, scenario H arises if $b_1 > c_1$ and $c_0 > b_0$, so that $s^f = \frac{c_0 - b_0}{b_1 - c_1}$, and $\overrightarrow{\alpha} \equiv c_1/b_1$. If $\alpha > \overrightarrow{\alpha}$ then scenario D arises and $s^* = \overline{s}$. If $\alpha < c_1/b_1$ then scenario I arises and $s^* = \frac{c_1+2\alpha b_0}{c_1-2\alpha b_1}$ if $0 < \alpha < \hat{\alpha} = \frac{c_1}{2(b_0+b_1)}$ and $s^* = \overline{s}$ if $\hat{\alpha} < \alpha < \overrightarrow{\alpha}$ where $\overrightarrow{\alpha} > \hat{\alpha}$. To draw Figure 4(c) to 4(e) we have used the following parameter values: $b_0 = b_1 = 2$, $c_1 = 1$, $c_0 = 2.5$, $\underline{s} = 0$ so that $s^f = 0.5$, $\hat{\alpha} = 0.125$, $\overrightarrow{\alpha} = 0.5$. We obtain $s^* = \frac{4\alpha}{1-4\alpha}$ if $0 < \alpha < 0.125$, and $s^* = \overline{s}$ if $\alpha > 0.125$. The three figures correspond to $\alpha \in \{0.05, 0.1, 0.2\}$ with $s^* \in \{0.25, 0.67, 1\}$.

Also for this scenario, it is straightforward to consider corner solutions. It is optimal to treat all patients if $c(s) - b(s) < 0$. For sufficiently low altruism, the DRG system will not induce this result. Individuals with high severity will be dumped. Again, it can happen that a DRG is active even though nobody should be treated. This case applies if $c(\overline{s}) - b(\overline{s}) > 0$. Providers may nevertheless treat patients. Consider the case of perfect altruism with $\alpha = 1$. Then scenario D obtains. If the initial price is at least $p^D_{min} = c(\overline{s}) - b(\overline{s})$, then an equilibrium will be reached in which all are treated even though $b(s) < c(s)$ for all patients.

### 3.3 Discussion

For both scenarios, we can conclude that the DRG price system with prices based on lagged average cost cannot be expected to give providers the incentive to treat the “right” patients. In particular, this applies for situations in which only a fraction of patients should be treated.
Only by chance will providers treat only those patients whose benefits exceed costs. When only high-severity patients should be treated, the optimal solution can never be implemented.

From a regulatory perspective, corner solutions are not as problematic as interior solutions. If nobody should be treated, the regulator could simply eliminate a DRG. If all patients should be treated, altruism may be sufficiently high to induce providers to treat all. Furthermore, dumping can be severely punished. For interior solutions, by contrast, some patient selection is optimal and it is difficult for the payer to infer whether the right patients are treated.

To cope with the problem of wrong incentives, two approaches can be taken. First, incentives could be improved by deviating from the average cost rule. For example, by setting the price to $p_f = c(s_f) - \alpha b(s_f)$ in Scenario L, the first-best treatment decision can be induced. However, hospitals costs and revenue will then differ. Therefore, hospitals must either be given a transfer (if $s_f < s^*$ requiring $p_f < p^*$) or must pay back a lump-sum (if $s_f > s^*$ requiring $p_f > p^*$). Especially the latter policy may be difficult to implement. Furthermore, this approach does not work in Scenario H if providers have a preference for low-severity patients. Then the price $p_f$ would induce provider to treat exactly the wrong patients.

The second approach is to separate treatment and rationing. This requires an agent who assigns patients for treatment exactly if benefits exceed costs. However, it is difficult to find such an institution. Physicians who send their patients to the hospital do not qualify because they can also be expected to have altruistic concerns and therefore are likely to support treatment whenever patient benefit is positive. Consulting neutral experts is an option. However, this causes additional expenditure and may delay treatment.

4 Conclusions

This study has investigated hospitals' dynamic incentives to select patients under a range of assumptions regarding the degree of providers' altruism and patients' benefit and cost functions. Since hospitals' prices are based on lagged average cost, a dynamic framework is essential to understand providers' incentives.

We showed that hospitals' dynamic incentives to select patients depend on the degree of providers' altruism and the relation between patients' severity and its benefit. For sufficiently
low altruism, the incentive to avoid expensive patients with high severity may be reinforced over time until a steady-state is reached, leading to a sort of spiral to the bottom. The introduction of a DRG price system generates incentives to dump patients which implies an even lower price in the future period which further encourages selection and dumping and so on. In the steady-state equilibrium, severity is such that the marginal benefit from altruistic concerns equates the difference between the marginal and average cost of treatment. This relation implies that higher altruism leads to higher equilibrium prices. This is in sharp contrast with what obtained under optimal pricing rules where the optimal price reduces with altruism (Ellis and McGuire, 1986).

An upward spiral of prices is also possible. Altruistic providers may treat patients whose costs are much larger than the DRG price. This in turn implies with a delay a higher DRG tariff which encourages providers to treat further high-cost patients. This mechanism can reinforce an equilibrium with no dumping. If altruism is high and patient benefit increases with severity, another pattern can arise. Providers prefer to treat high-severity patients and prices can then first rise. This creates incentives to extend treatment to low-severity patients and prices fall again.

Welfare implications of current DRG systems have also been derived. We assumed that patients should be treated if benefits exceed costs and compared this to the equilibrium of the DRG system. We found that the optimal treatment decision is only implemented by chance and that severe deviations can arise. In particular, this holds for low altruism. Then DRG pricing may lead to too few patients being treated if patients’ benefit decreases with severity. When patients’ benefit increases with severity (for example higher severity causes higher pain without treatment), then in the DRG pricing may give completely the wrong incentives. For low altruism, providers have incentive to treat low-severity patients while it would be optimal from a first-best perspective to treat high-severity ones. For high altruism, the problem is overtreatment, either of high-severity or of low-severity patients.

Overall, our analysis shows that a DRG system in which prices are calculated according to the average-cost pricing rules cannot be expected to give the right incentives to treat patients. Over time, price dynamics can lead to undesirable outcomes. This holds in particular for treatments which are justified only for fraction of patients either because costs are too high or benefits
too low for some patients. An example are complicated surgeries which can be very costly for patients in otherwise poor health state and should therefore be limited to individuals with low severity. On the other hand, some treatments should be applied only to high-severity individuals if they benefit most.

Our key assumption is that hospitals decide about treatment. In practice, they are limitations to this rule when it comes to the “dumping” of patients. For example, patients may protest against not receiving treatment. Nevertheless, patients can be rejected by stating that a treatment has little chance of success or by pretending to be operating above capacity. With respect to overtreatment, hospitals can be expected to meet less resistance. In our model, we assumed that all patients receive positive benefits from treatment and are therefore unlikely to protest if they get treated even though costs exceed benefits.

Our model made a number of simplifying assumptions. Providers were assumed to be homogeneous. There was only one DRG. We also have not considered any capacity constraints which could prevent hospitals from treating too many cases. Further research could relax these assumptions in a dynamic framework. Our results could also be useful for empirical studies of DRG data to detect patterns of patient selection by hospitals. Finally, future work could examine modifications of the DRG system to give providers better incentives to treat exactly those whose benefits exceed costs.
References


Appendix

A.1 Comparative statics with respect to $\alpha$

Differentiating (10) with respect to $\alpha$, we obtain

$$\frac{\partial s^*}{\partial \alpha} = \frac{b(s^*)}{-\left[\alpha b'(s^*) - c'(s^*) + \frac{f(s^*)}{F(s^*)} \left( c(s^*) - \frac{\int_0^{s^*} c(s)f(s)ds}{F(s^*)} \right) \right]}.$$

The stability condition requires

$$\frac{dp_t}{dp_{t-1}} \bigg|_{ss} = \frac{f(s^*)}{F(s^*)} \left[ c(s^*) - \frac{\int_0^{s^*} c(s)f(s)ds}{F(s^*)} \right] - \frac{1}{-\left[\alpha b'(s) - c'(s)\right]} < 1,$$

which implies that the denominator in $\frac{\partial s^*}{\partial \alpha}$ is positive. Moreover,

$$\frac{\partial p^*}{\partial \alpha} = \frac{f(s^*)}{F(s^*)} \left( c(s^*) - \frac{\int_0^{s^*} c(s)f(s)ds}{F(s^*)} \right) \frac{\partial s^*}{\partial \alpha} > 0.$$

A.2 Example

In our example, benefits and costs are linear in severity: $b(s) = b_0 + b_1 s$, $c(s) = c_0 + c_1 s$ with $c_0 > 0$, $c_1 > 0$, $b_0 > 0$. Patients’ severity is distributed over the support $[\underline{s}, \bar{s}]$ with the uniform density function with a mass of one, $f(s) = 1$, (implying $\bar{s} - \underline{s} = 1$).

Scenario I: We have $c'(s) - \alpha b'(s) > 0$ and therefore $c_1 - \alpha b_1 > 0$. This is always satisfied when $b_1 < 0$ but also if $b_1 > 0$ and altruism is sufficiently low, ie $\alpha < \bar{\alpha} \equiv c_1/b_1$. Condition (1) which defines the marginal treated patients corresponds to

$$p_t = c_0 + c_1 s_t - \alpha \left( b_0 + b_1 s_t \right),$$

implying $s_t = \frac{p_t + \alpha b_0 - c_0}{c_1 - \alpha b_1}$.

Using (2), DRG pricing implies $p_t = c_0 + \frac{c_1 (s_{t-1} + \bar{s})}{2}$. Substituting $s_{t-1}$,
we obtain the linear function:

$$g(p_t) = \frac{A}{2(c_1 - \alpha b_1)} + \frac{c_1}{2(c_1 - \alpha b_1)}p_{t-1}$$

with $A \equiv sc_1(c_1 - \alpha b_1) + c_0c_1 + \alpha(c_1b_0 - 2b_1c_0)$. Moreover, $p^f_{\min} \equiv (c_0 + c_1s) - \alpha(b_0 + b_1s)$, $p^f_{\text{all}} \equiv (c_0 + c_1s) - \alpha(b_0 + b_1s)$ and $c^\mu \equiv c_0 + c_1\frac{(s + \bar{s})}{2}$. Due to the linearity, there is always a unique equilibrium for $p_0 \geq p^f_{\min}$. For the interior equilibrium which holds for $0 < \alpha < \hat{\alpha} = c_1/[2(b_0 + b_1\bar{s})]$, we obtain $p^* = \frac{A}{c_1 - 2\alpha b_1}$, $s^* = \frac{sc_1 + 2\alpha b_0}{c_1 - 2\alpha b_1}$.

**Scenario D:** This scenario arises if $\alpha b_1 > c_1$ or, equivalently, $\alpha > \tilde{\alpha} \equiv c_1/b_1$. Patients are treated if $\alpha(b_0 + b_1s_t) + p_t \geq c_0 + cs_t$, leading to $s_t = \frac{c_0 - \alpha b_0 - p_t}{\alpha b_1 - c_1}$. Using (11), DRG pricing implies $p_t = c_0 + \frac{c_1(s + s_t - 1)}{2}$. Thus, we obtain the linear function $h(p_{t-1}) = c_0 + \frac{c_1s}{2} + \frac{c_1(c_0 - \alpha b_0)}{2(\alpha b_1 - c_1)} - \frac{c_1}{2(\alpha b_1 - c_1)}p_{t-1}$, with $p^D_{\min} = c_0 + c_1s - \alpha b_0 - \alpha b_1s$, $p^D_{\text{all}} = c_0 + c_1s - \alpha b_0 - \alpha b_1s$, and $c^\mu \equiv c_0 + c_1\frac{(s + \bar{s})}{2}$. In equilibrium, we have a corner solution with $s^* = \bar{s}$ and $p^* = c_0 + \frac{c_1(s + \bar{s})}{2}$.

Noting that $\tilde{\alpha} = \frac{c_1}{b_1} > \hat{\alpha} = \frac{c_1}{2(\alpha b_0 + \alpha b_1\bar{s})}$, all patients are treated in this equilibrium.
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Esplanade 36
20354 Hamburg
Germany
Tel: +49 (0) 42838-8041
Fax: +49 (0) 42838-8043
Email: info@hche.de
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